

Transpose of a Matrix (A^T or A')

* Interchange the rows & columns

* Eg: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

Matrix Multiplication

* $O(A) = m \times n$
 * $O(B) = n \times p$ } $O(AB) = m \times p$

* If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$

then

$$AB = \begin{bmatrix} 3+0+3 & 1+4+9 \\ 6+0+4 & 2+2+12 \end{bmatrix} = \begin{bmatrix} 9 & 14 \\ 10 & 16 \end{bmatrix}$$

Properties

- ① $(A')' = A$
- ② $(kA)^T = k(A^T)$
- ③ $(A+B)^T = A^T + B^T$
- ④ $(AB)^T = B^T A^T$
- ⑤ Symmetric : $A^T = A$
- ⑥ Skew symmetric : $A^T = -A$

* Note: For a skew symmetric elements in the leading diagonal is always zero.

EXERCISE 3.3

1. Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

(i) Given $A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) $B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

2. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

(i) $(A+B)' = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

(ii) $A-B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1+4 & 2-1 & 3+5 \\ 5-1 & 7-2 & 9-0 \\ -2-1 & 1-3 & 1-1 \end{bmatrix}$

$$A-B = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$(A-B)^T = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \longrightarrow \textcircled{1}$$

$$A^T - B^T = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \longrightarrow \textcircled{2}$$

From $\textcircled{1}$ & $\textcircled{2}$ $(A-B)^T = A^T - B^T$

3. If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

— Do —